Прелиставање ...
Knuth
THE ART OF PROGRAMMING

The **dictionary** by Merriam-**Webster**

preliminary
adjective
Definition of *preliminary* : coming before and usually forming a necessary prelude to something else*preliminary* studies*preliminary* results

History and Etymology for *preliminary*
Noun
French *préliminaires*, plural, from Medieval Latin *praeliminaris*, adjective, preliminary, from Latin *prae-* pre- + *limin-*, *limen* threshold

Математичке прелиминарије

- Математичка индукција

1.2.1. Mathematical Induction

Let $P(n)$ be some statement about the integer n; for example, $P(n)$ might be "n times $(n+3)$ is an even number," or "if $n \geq 10$, then $2^n > n^3$." Suppose we want to prove that *$P(n)$ is true for all positive integers n.* An important way to do this is:

a) Give a proof that $P(1)$ is true.

b) Give a proof that "if all of $P(1), P(2), \ldots, P(n)$ are true, then $P(n+1)$ is also true"; this proof should be valid for any positive integer n.

Преузето са https://doc.lagout.org/science/0_Computer%20Science/2_Algorithms/The%20Art%20of%20Computer%20Programming%20(vol.%201_%20Fundamental%20Algorithms)%20(3rd%20ed.)%20%5BKnuth%201997-07-17%5D.pdf

As an example, consider the following series of equations, which many people have discovered independently since ancient times:

$$\begin{aligned} 1 &= 1^2, \\ 1+3 &= 2^2, \\ 1+3+5 &= 3^2, \\ 1+3+5+7 &= 4^2, \\ 1+3+5+7+9 &= 5^2. \end{aligned} \tag{1}$$

We can formulate the general property as follows:

$$1+3+\cdots+(2n-1)=n^2. \tag{2}$$

Доказујемо:

1. p(1) је тачно.
2. Импликација p(n) → p(n+1) је тачна за сваки позитиван цео број n.

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   $1 + 3 + \ldots + (2\mathrm{n} - 1) = n^2.$
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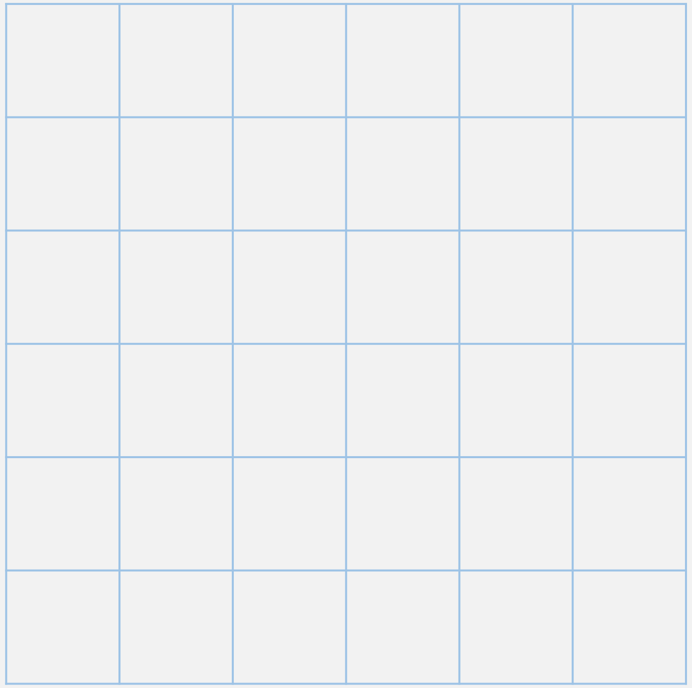
   $n^2 + 2n + 1 = (n + 1)^2$, qed.

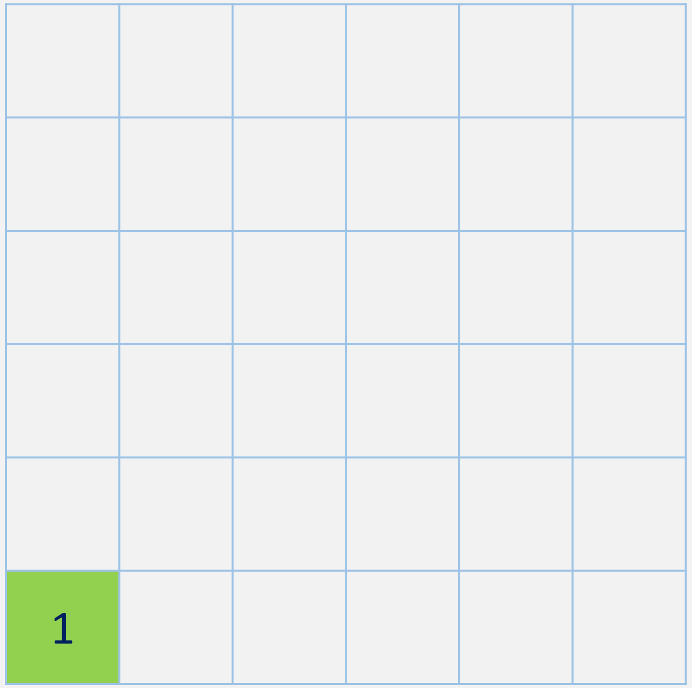
Геометријска интерпретација једнакости

$1 + 3 + \ldots + (2n - 1) = n^2$ за $n = 6$.



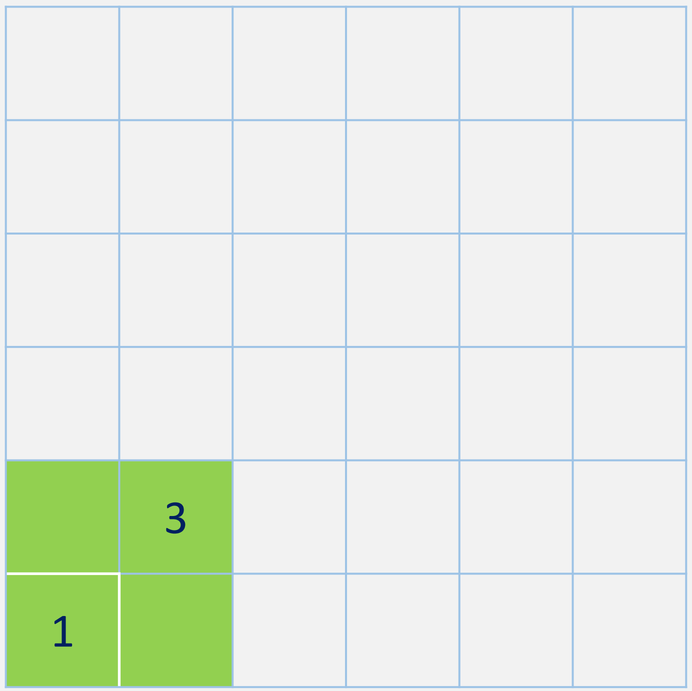
Геометријска инерпретација једнакости

$$1 = 1^2.$$



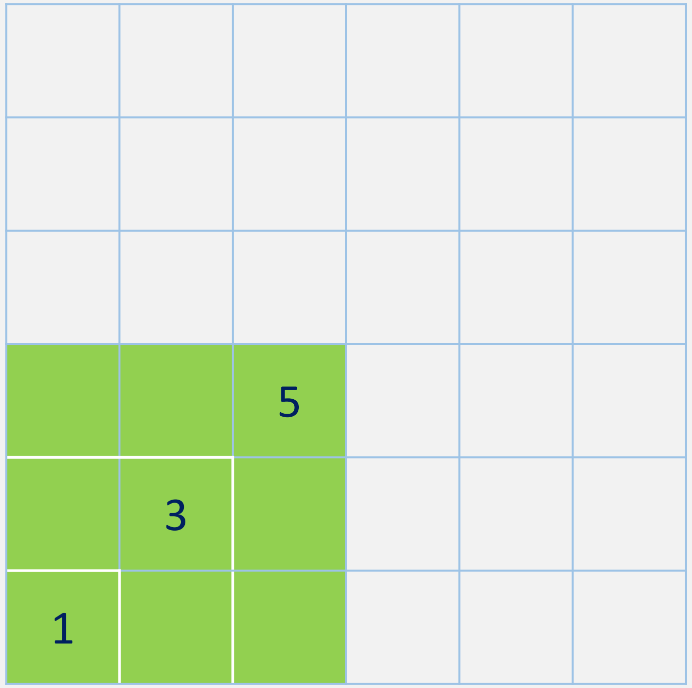
Геометријска инерпретација једнакости

$$1 + 3 = 2^2.$$



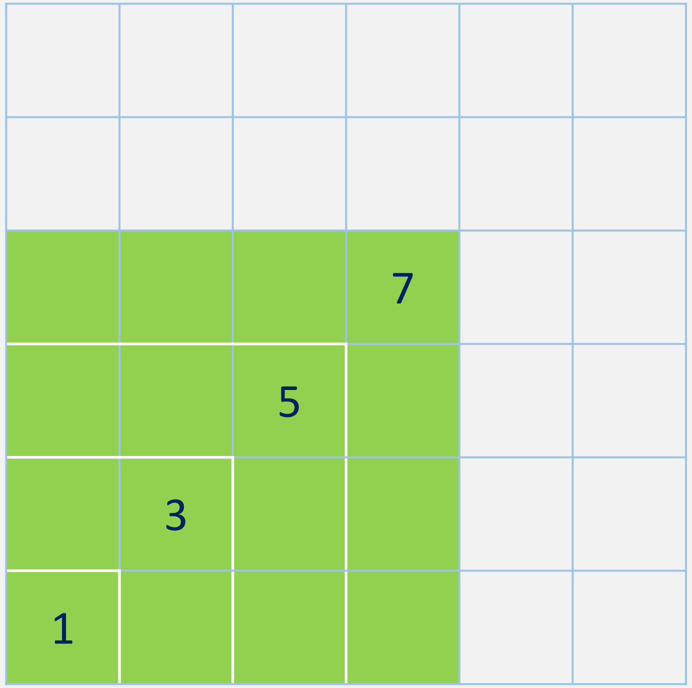
Геометријска инерпретација једнакости

$$1 + 3 + 5 = 3^2.$$



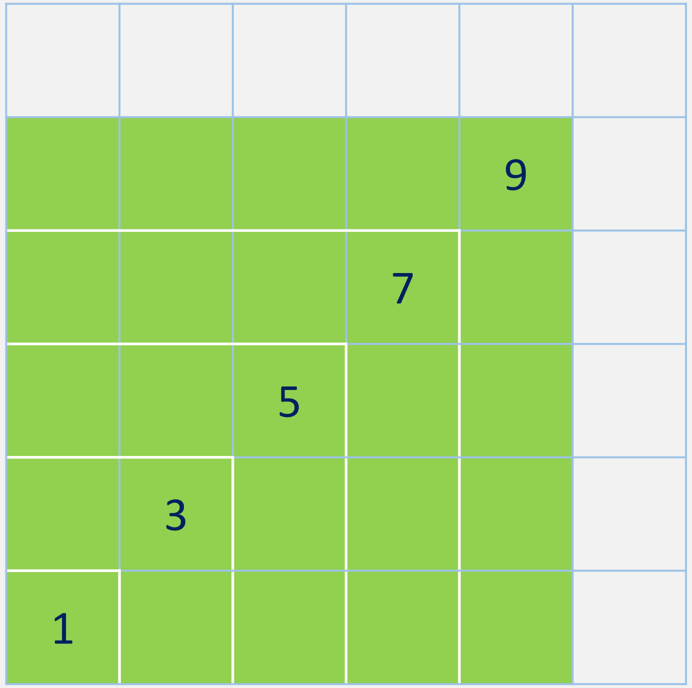
Геометријска инерпретација једнакости

$$1 + 3 + 5 + 7 = 4^2.$$



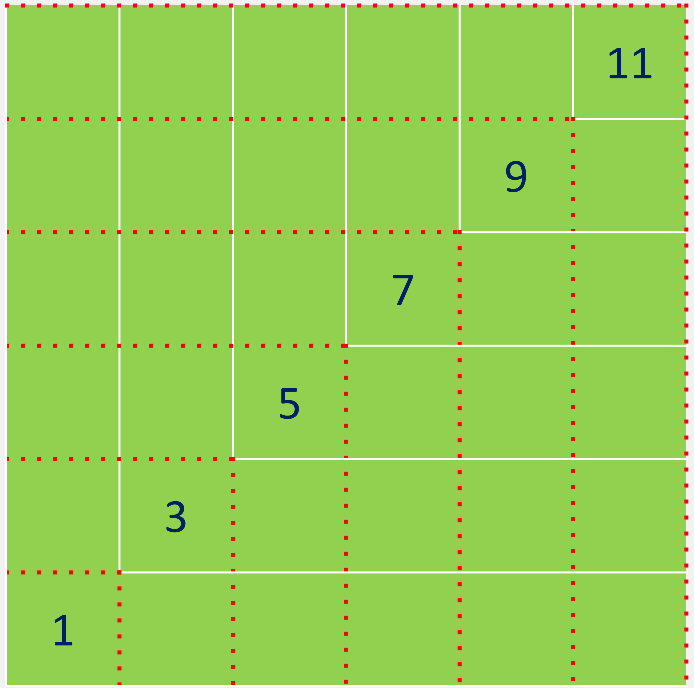
Геометријска инерпретација једнакости

$$1 + 3 + 5 + 7 + 9 = 5^2.$$



Геометријска инерпретација једнакости

$$1 + 3 + 5 + 7 + 9 + 11 = 6^2.$$



6. [*20*] Prove that if Eqs. (6) hold before step E4 is performed, they hold afterwards also.

7. [*23*] Formulate and prove by induction a rule for the sums 1^2, $2^2 - 1^2$, $3^2 - 2^2 + 1^2$, $4^2 - 3^2 + 2^2 - 1^2$, $5^2 - 4^2 + 3^2 - 2^2 + 1^2$, etc.

▶ **8.** [*25*] (a) Prove the following theorem of Nicomachus (A.D. c. 100) by induction: $1^3 = 1$, $2^3 = 3 + 5$, $3^3 = 7 + 9 + 11$, $4^3 = 13 + 15 + 17 + 19$, etc. (b) Use this result to prove the remarkable formula $1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$.

[*Note:* An attractive geometric interpretation of this formula, suggested to the author by R. W. Floyd, is shown in Fig. 5. The idea is related to Nicomachus's theorem and Fig. 3. Other "look-see" proofs can be found in books by Martin Gardner, *Knotted Doughnuts* (New York: Freeman, 1986), Chapter 16; J. H. Conway and R. K. Guy, *The Book of Numbers* (New York: Copernicus, 1996), Chapter 2.]

$$\text{Side} = 5+5+5+5+5+5 = 5\cdot(5+1)$$

$$\text{Side} = 5+4+3+2+1+1+2+3+4+5 = 2(1+2+\cdots+5)$$

$$\text{Area} = 4\cdot 1^2 + 4\cdot 2\cdot 2^2 + 4\cdot 3\cdot 3^2 + 4\cdot 5\cdot 5^2 = 4(1^3+2^3+\cdots+5^3)$$

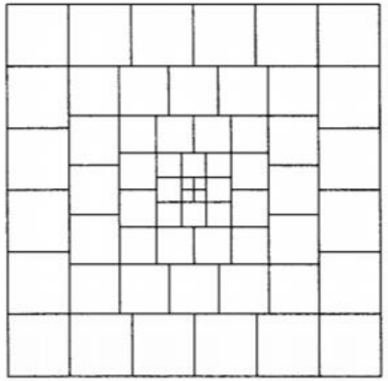


Fig. 5. Geometric version of exercise 8(b).

9. [*20*] Prove by induction that if $0 < a < 1$, then $(1-a)^n \geq 1 - na$.

10. [*M22*] Prove by induction that if $n \geq 10$, then $2^n > n^3$.

11. [*M30*] Find and prove a simple formula for the sum

$$\frac{1^3}{} - \frac{3^3}{} + \frac{5^3}{} - \cdots + \frac{(-1)^n(2n+1)^3}{}.$$

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▸ **8.** [*25*] (a) Prove the following theorem of Nicomachus (A.D. c. 100) by induction: $1^3 = 1$, $2^3 = 3 + 5$, $3^3 = 7 + 9 + 11$, $4^3 = 13 + 15 + 17 + 19$, etc. (b) Use this result to prove the remarkable formula $1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$.

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(a)

1.

$a_1 = 1 = 1^3$;

2.

$a_n = n(n-1) + 1 + n(n-1) + 3 + \ldots + n(n-1) + 2n - 1 = n^3$;

$a_{n+1} = n(n-1) + 2n + 1 + n(n-1) + 2n + 3 + \ldots + n(n-1) + 2n + 2n - 1 + n(n-1) + 2n + 2n + 1 =$

$n(n-1) + 2n + 1 + n(n-1) + 2n + 3 + \ldots + n(n-1) + 2n + 2n - 1 + n(n-1) + 2n + 2n + 1 =$

$n^3 + 2n(n+1) + n(n-1) + 2n + 2n + 1 =$

$n^3 + 2n^2 + 2n + n^2 - n + 2n + 1 =$

$n^3 + 3n^2 + 3n + 1 =$

$(n+1)^3$, qed.

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(b)

$1^3 = 1,$
$2^3 = 3 + 5,$
$3^3 = 7 + 9 + 11,$
$4^3 = 13 + 15 + 17 + 19,$
$\ldots,$
$n^3 = n(n-1) + 1 + n(n-1) + 3 + \ldots + n(n-1) + 2n - 1.$

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(b)

$$1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 =$$

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + \ldots + n(n-1) + 1 + n(n-1) + 3 + \ldots + n(n-1) + 2n - 1 =$$

$$\frac{\frac{n(n+1)}{2}}{2} \cdot (1 + n(n-1) + 2n - 1) = \left(\frac{n(n+1)}{2}\right)^2.$$

$$(1 + 2 + \ldots + n)^2 = \left(\frac{n(n+1)}{2}\right)^2.$$

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(b)

$$1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 =$$

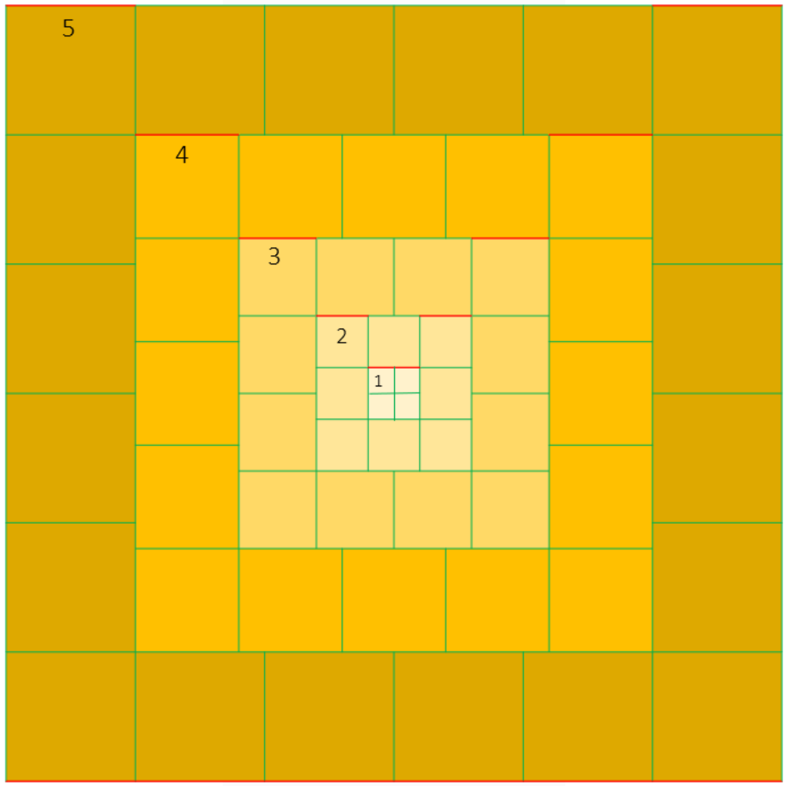
$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + \ldots + n(n-1) + 1 + n(n-1) + 3 + \ldots + n(n-1) + 2n - 1 =$$

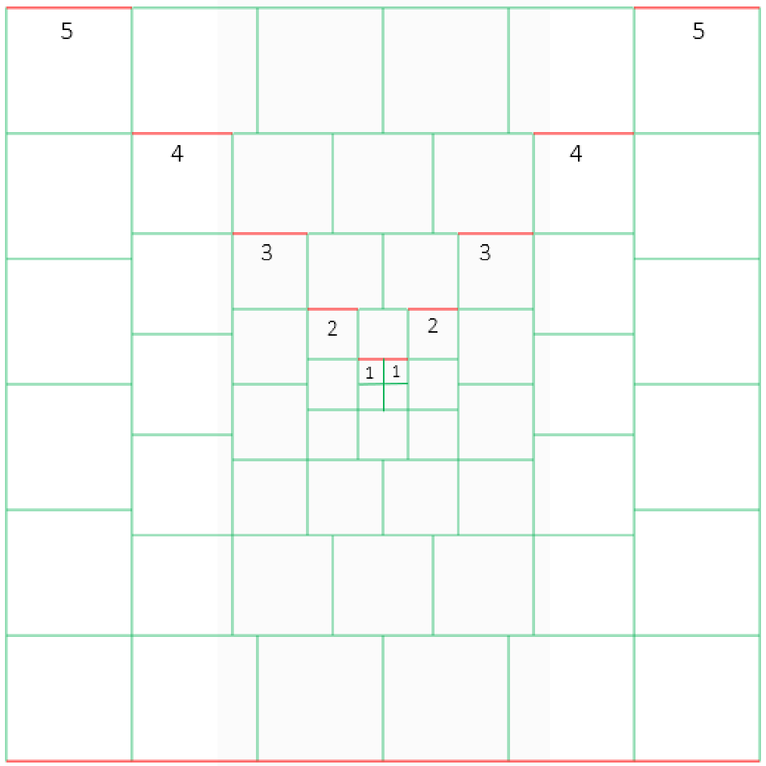
$$\frac{\frac{n(n+1)}{2}}{2} \cdot (1 + n(n-1) + 2n - 1) = \left(\frac{n(n+1)}{2}\right)^2 .$$

$$(1 + 2 + \ldots + n)^2 = \left(\frac{n(n+1)}{2}\right)^2 .$$

→ $1^3 + 2^3 + \ldots + n^3 = (1 + 2 + \ldots + n)^2$.

Геометријска интерпретација





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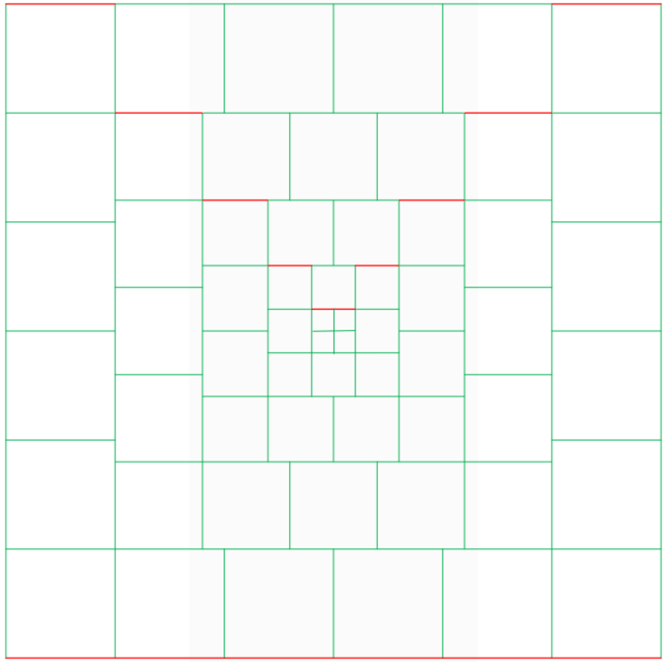

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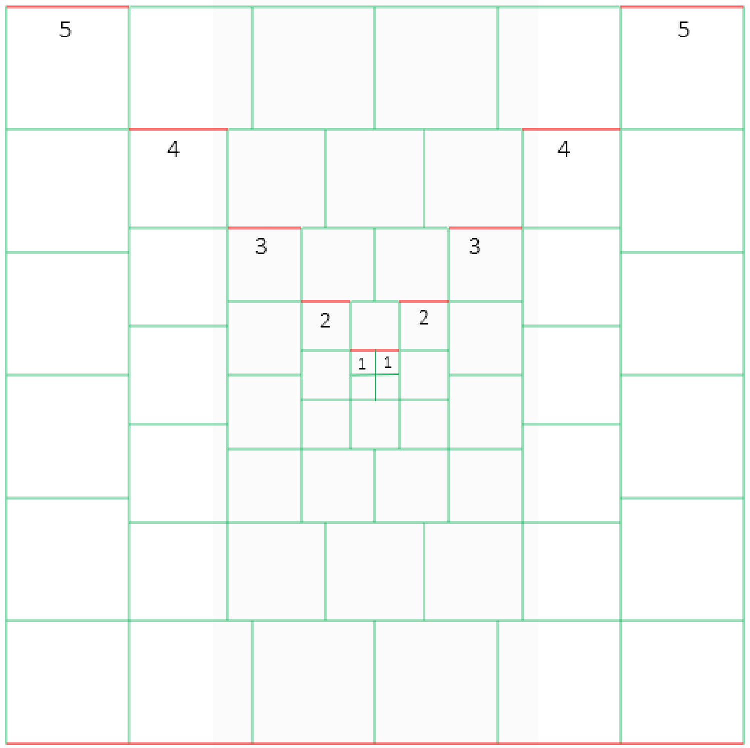

- Страница квадрата =

  5 + 5 + 5 + 5 + 5 + 5 = 5 · (5 + 1);

- Страница квадрата =

  5 + 4 + 3 + 2 + 1 + 1 + 2 + 3 + 4 + 5 =

  2 · (1 + 2 + 3 + 4 + 5);

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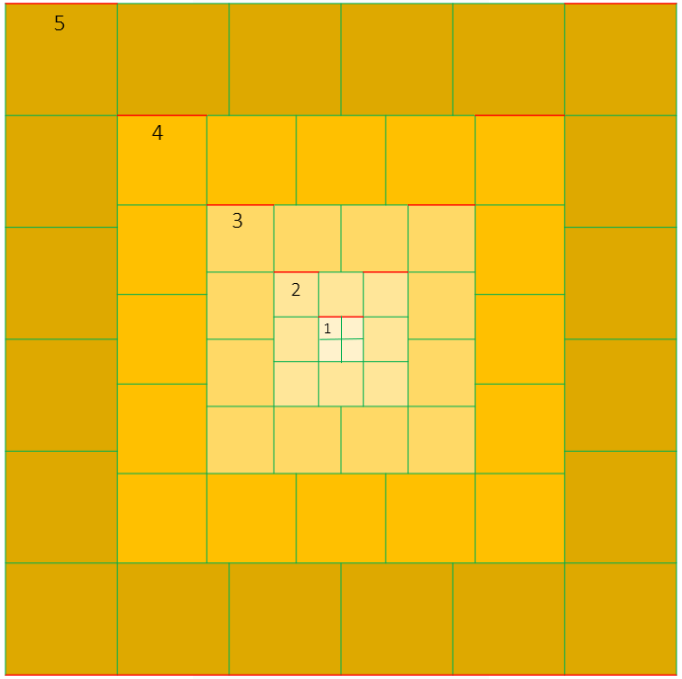

- Страница квадрата =

  $5 + 4 + 3 + 2 + 1 + 1 + 2 + 3 + 4 + 5 =$

  $2 \cdot (1 + 2 + 3 + 4 + 5);$

- Површина квадрата =

  $2 \cdot (1 + 2 + 3 + 4 + 5) \cdot 2 \cdot (1 + 2 + 3 + 4 + 5) =$

  $4 \cdot (1 + 2 + 3 + 4 + 5)^2.$

- Површина квадрата =

  $4 \cdot 1^2 + 4 \cdot 2 \cdot 2^2 + 4 \cdot 3 \cdot 3^2 + 4 \cdot 4 \cdot 4^2 + 4 \cdot 5 \cdot 5^2 =$

  $4 \cdot (1^3 + 2^3 + 3^3 + 4^3 + 5^3);$

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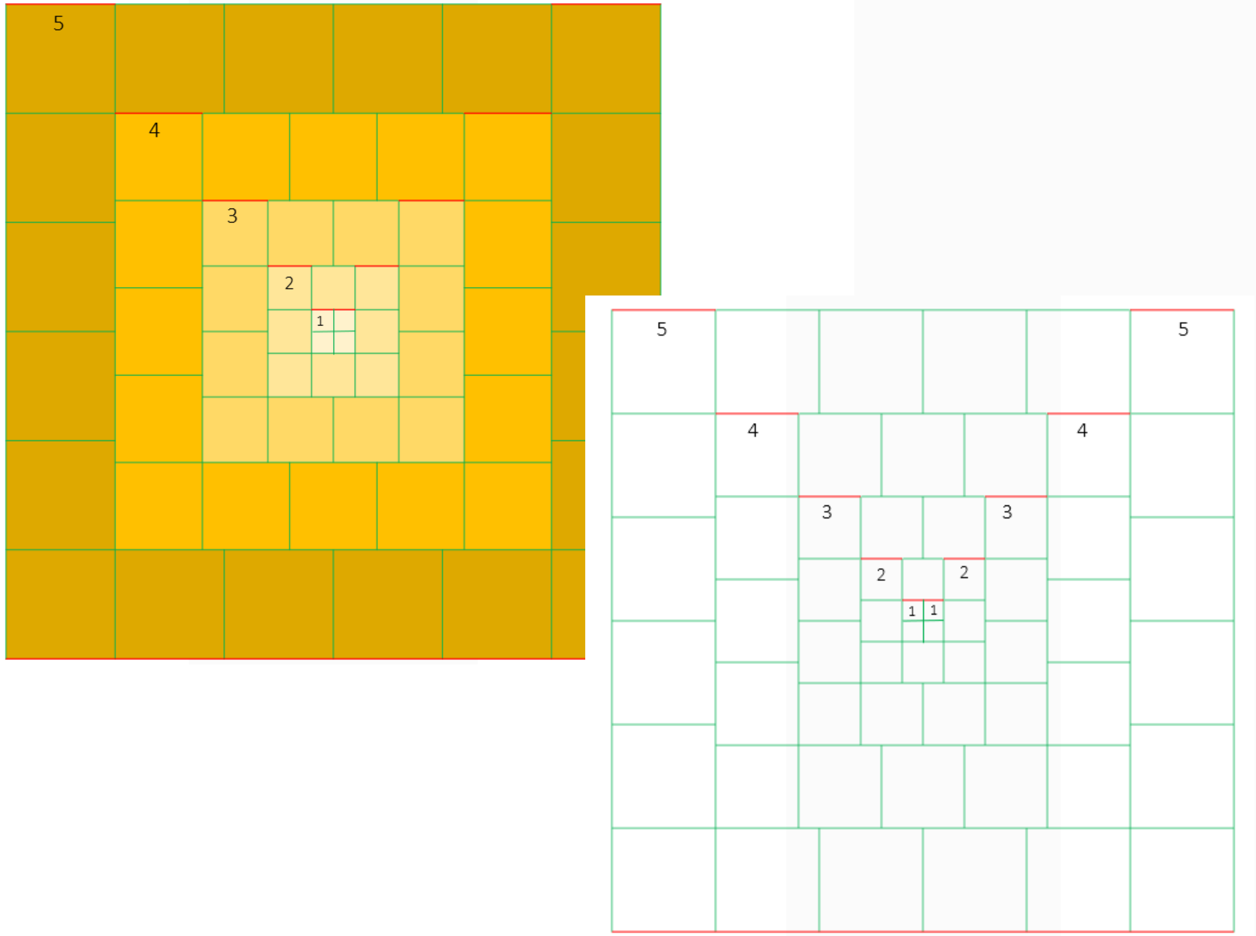

- Површина квадрата =

  $4 \cdot 1^2 + 4 \cdot 2 \cdot 2^2 + 4 \cdot 3 \cdot 3^2 + 4 \cdot 4 \cdot 4^2 + 4 \cdot 5 \cdot 5^2 =$

  $4 \cdot (1^3 + 2^3 + 3^3 + 4^3 + 5^3);$

- Површина квадрата =

  $2 \cdot (1 + 2 + 3 + 4 + 5) \cdot 2 \cdot (1 + 2 + 3 + 4 + 5) =$

  $4 \cdot (1 + 2 + 3 + 4 + 5)^2.$

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1 + 2 + 3 + 4 + 5)^2$$

- Наставак следи ...